Part V

Acceptance Sampling

Chapter 14

Lot–By–Lot Acceptance Sampling for Attributes

We look at lot–by–lot acceptance sampling plans for attributes.

14.1 The Acceptance–Sampling Problem

Often, lot–by-lot acceptance sampling plans are based on probability models and involve sampling at random from a lot of size n, and deciding, on the basis of the number of defectives, c, in the sample, to either accept or reject the lot. A number of different lot–by–lot acceptance sampling plans are discussed in this chapter.

1. *Single–sample* (from a particular lot) plans involve taking one (and only one) sample from a lot and using the number of defectives in this sample to decide either to accept or reject the entire lot.

2. *Multiple–sample* (from a particular lot) plans, particularly *double–sample plans*, involve, after every sample, deciding to either accept the lot, continue sampling, or reject the lot, until a fixed predetermined (finite) number of multiple samples has been taken, at which point, we must decide to either accept or reject the lot.

3. *Sequential sampling* plans are much like multiple–sample plans that involve, after every sample, deciding to either accept the lot, continue sampling, or reject the lot, but, unlike multiple–sample plans, there is no predetermined finite number of samples to be made.

4. *Standard sampling plans*, such as the MIL STD 105E and Dodge–Romig plans, that are only partially based on probability distributions, are also discussed.

14.2 Single–Sampling Plans For Attributes

SAS program: att13-14-2-books-oc,aoq,ati

Single–sample (from a particular lot) plans involve taking one (and only one) sample of size n from a lot of size N and using the number of defectives, c, in this sample to decide either to accept or reject the entire lot. We will use a number of techniques to help us in single–sample plans, including

1. operating characteristic (OC) curves
2. average outgoing quality (AOQ) or average fraction defective and the related average outgoing quality limit (AOQL).
3. average total inspection (ATI)
4. Binomial nomograph

Exercise 13.1 (Single–Sampling Plans For Attributes)

1. *Operating characteristic (OC) curve,* $n = 150$, $c = 3$
   A lot of $N = 3500$ books arrives at a bookstore. To check if the entire lot is acceptable, a random sample of $n = 150$ books is taken from the lot and if $c = 3$ or *less* of the books are found to be defective in any way (misprints, bad binding and so on), the entire lot is *accepted*.

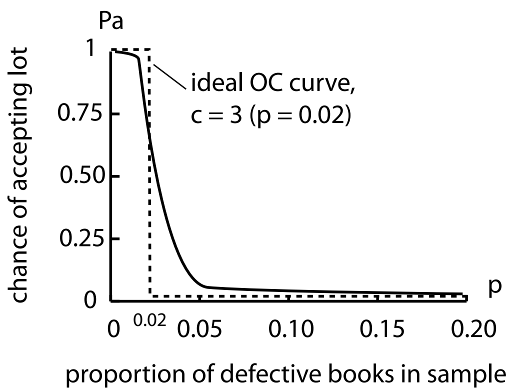

Figure 13.1 (Operating characteristic (OC) curve, $n = 150$, $c = 3$)

   (a) If four (4) defective books are found in the sample of $n = 150$, the entire lot of books is (choose one) **accepted** / **rejected**.
   (b) If one (1) defective book is found in the sample of $n = 150$, the entire lot of books is (choose one) **accepted** / **rejected**.

(c) According to the OC curve, the *chance* of accepting the lot is
P_a = (choose one) **0.75** / **0.95** / **1**
if there are (unknown to us) actually zero (0) defective books in the sample of $n = 150$.

(d) According to the OC curve, the *chance* of accepting the lot is
P_a = (choose one) **0.00012** / **0.00047** / **0.00171**
if there are (unknown to us) actually 15 defective books in the sample of $n = 150$ or, in other words, $p = \frac{15}{150} = 0.10$.

(e) According to the OC curve, the *chance* of accepting the lot is
P_a = (choose one) **0.657** / **0.935** / **1**
if there are (unknown to us) actually 3 defective books in the sample of $n = 150$ or, in other words, $p = \frac{3}{150} = 0.02$.

(f) An *ideal* OC curve is one where there is a 100% chance of accepting the lot that has 3 or fewer defective books and so a 0% chance of accepting a lot that has 4 or more defective books. The larger the sample size, n, the (choose one) **closer** / **farther** the OC curve will be (to) / (away from) the ideal OC curve.

(g) **True** / **False**
This OC curve is based on a binomial probability distribution and is given by calculating, for different $p = 0, 0.01, ...,$

$$P_a = \{d \leq c\} = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d} = \sum_{d=0}^{3} \frac{350!}{d!(350-d)!} p^d (1-p)^{350-d}$$

which, notice, does *not* depend on lot size, N ($N = 3500$, in this case). This is an example of a *type B OC curve*. *If* the OC curve was based on the hypergeometric distribution, it would be influenced by the lot size and be called a *type A OC curve*.

2. *Average outgoing quality (AOQ) curve*
   A lot of 3500 books arrives at a bookstore. To check if the entire lot is acceptable, a random sample of 150 books is taken from the lot and if $c = 3$ or *less* of the books are found to be defective, the entire lot is *accepted*.

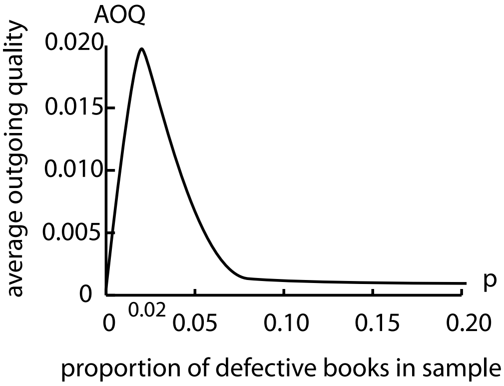

Figure 13.2 (AOQ curve, $N = 3500$, $n = 150$, $c = 3$)

   (a) The average outgoing quality limit (AOQL) or, in other words, the worse average fraction defective occurs at
   (choose one) $\boldsymbol{p = 0.01}$ / $\boldsymbol{p = 0.02}$ / $\boldsymbol{p = 0.03}$
   and is given by AOQL = 0.01239. This means, that *on average*, a typical lot will have a fraction defective no worse than 1.23% given the sampling plan of $n = 150$ and $c = 3$.

   (b) The AOQ curve is given by calculating, for different $p = 0, 0.01, ...,$

$$\text{AOQ} = \frac{P_a p(N-n)}{N} = \frac{P_a p(3500-150)}{3500}$$

   For example, at $p = 0.02$, we determine that $P_a = 0.64724$ and so

$$\text{AOQ} = \frac{P_a p(N-n)}{N} = \frac{(0.64724)(0.02)(3500-150)}{3500} =$$

   0.01112 / 0.01239 / 0.03223
   This is not too surprising, since this is the AOQL.

3. *Average total inspection (ATI) curve*
   A lot of 3500 books arrives at a bookstore. To check if the entire lot is acceptable, a random sample of 150 books is taken from the lot and if $c = 3$ or *less* of the books are found to be defective, the entire lot is *accepted.*

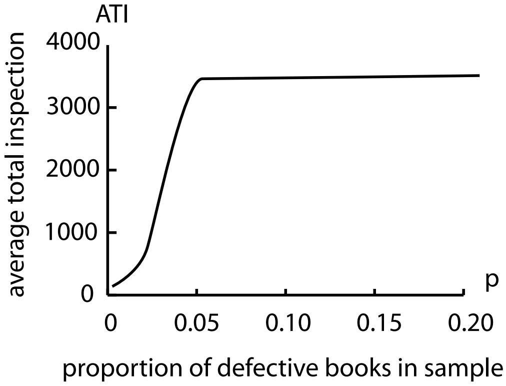

Figure 13.3 (ATI curve, $N = 3500$, $n = 150$, $c = 3$)

(a) On average, essentially *all* of the books in the entire lot need to be inspected *beginning* at an actual proportion defective of roughly (choose one) **0.01 / 0.06 / 0.15**

(b) The ATI per lot curve is given by calculating, for different $p = 0, 0.01, ...,$

$$\text{ATI} = n + (1 - P_a)(N - n) = 150 + (1 - P_a)(3500 - 150)$$

For example, at $p = 0.02$, we determine that $P_a = 0.64724$ and so

$$\text{ATI} = n + (1 - P_a)(N - n) = 150 + (1 - 0.64724)(3500 - 150)$$

(choose one) **1331.75 / 1567.77 / 2333.34**

4. *Binomial nomograph*
   The binomial nomograph is used to determine the sample size, n, and acceptance number, c, given the probability of acceptance is $1-\alpha$ for lots with a fraction p_1 defective and also the probability of acceptance is β for lots with fraction p_2 defective.

   (a) *A first example*
   From the binomial nomograph, page 690,
   when $p_1 = 0.01$, $1-\alpha = 1-0.05 = 0.95$, $p_2 = 0.04$ and $\beta = 0.20$,
   it appears that the sampling plan should be (choose one)

   i. $n \approx 50$ and $c \approx 1$
   ii. $n \approx 150$ and $c \approx 3$
   iii. $n \approx 500$ and $c \approx 7$

   Hint: Draw two lines between the two vertical scales on either side of the binomial nomograph. One line starts on the left scale at $p_1 = 0.01$ and ends at $1-\alpha = 0.95$ on the right scale. The other line starts on the left scale at $p_2 = 0.04$ and ends at $\beta = 0.20$ on the right scale. The n and c are read from where the two drawn lines meet.

   (b) *Verifying the binomial nomograph using the binomial formulas*
   Using the binomial formulas[1] to verify this plan where $n = 150$ and $c = 3$,

$$\begin{aligned}
1-\alpha &= \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_1^d (1-p_1)^{n-d} \\
&= \sum_{d=0}^{3} \frac{150!}{d!(150-d)!} 0.01^d (1-0.01)^{150-d} = 0.9353 \\
\beta &= \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_2^d (1-p_2)^{n-d} \\
&= \sum_{d=0}^{3} \frac{150!}{d!(150-d)!} 0.04^d (1-0.04)^{150-d} =
\end{aligned}$$

   (choose one) **0.1458** / **0.1758** / **0.1958**
   In other words, since $1-\alpha$ and β are (fairly) close to what they should be ($0.95 \approx 0.9353$ and $0.20 \approx 0.1558$, respectively), the binomial nomograph gives values of n and c close to what they should be.

   (c) *Understanding the binomial nomograph*
   True / **False**
   The $1-\alpha$ is the chance of (correctly) accepting the null (of accepting the

[1] 2nd DISTR binomcdf...

lot) and β is the chance of (mistakenly) accepting the lot. Both can be calculated using the given (known) fraction defective, p_1 and p_2, and the unknown n and c. Since there are two *nonlinear* equations, the two unknowns, n and c, can be often be determined, but not in a simple analytical way. Hence, the binomial nomograph.

14.3 Double, Multiple and Sequential Sampling

SAS program: att13-14-3-books-doublesample

Multiple–sample (from a particular lot) plans, particularly *double–sample plans*, involve, after every sample, deciding to either accept the lot, continue sampling, or reject the lot, until a fixed predetermined (finite) number of multiple samples has been taken, at which point, we must decide to either accept or reject the lot. *Sequential sampling* plans are much like multiple–sample plans that involve, after every sample, deciding to either accept the lot, continue sampling, or reject the lot, but, unlike multiple–sample plans, there is no predetermined finite number of samples to be made.

Exercise 13.2 (Double, Multiple and Sequential Sampling)

1. *Double sampling: books*
   A lot of 3500 books arrives at a bookstore. To check if the entire lot is acceptable, a *first* random sample of $n_1 = 150$ books is taken from the lot and if $c_1 = 1$ or less of the books are found to be defective, the entire lot is *accepted*; if greater than $c_2 = 5$ books are found, the lot is rejected. If between 2 and 4 books are found to be defective on the first sample, a *second* sample is taken of $n_2 = 100$ books and if the *combined* number of defects of the first and second samples is less than $c_2 = 5$, the lot is accepted and rejected otherwise.

   (a) *Probability of acceptance, first sample*
   The probability of accepting the first sample, of observing $d_1 \leq c_1 = 1$, for different $p = 0, 0.01, ...$, is

$$P_a^I = \{d_1 \leq c_1\} = \sum_{d_1=0}^{c_1} \frac{n_1!}{d_1!(n_1 - d_1)!} p^{d_1} (1-p)^{n_1-d_1} = \sum_{d_1=0}^{1} \frac{150!}{d_1!(150 - d_1)!} p^{d_1} (1-p)^{150-d_1}$$

   For example, at $p = 0.03$,
   $P_a^I =$ (choose one) **0.05848 / 0.0758 / 0.0958**

(b) *Probability of rejection, first sample*
True / False
The probability of *rejecting* the *first* sample, of observing $d_1 > c_1 = 1$, for different $p = 0, 0.01, ...,$ is

$$P_r^I = 1 - P_a^I$$

(c) *Probability of acceptance, second sample*
A *second* sample is required if the number of defectives found in the *first* sample, d_1, fall in the range, $c_1 < d_1 \leq c_2$. The second sample is accepted if the number of defectives found in the *combined* first and second samples, $d_1 + d_2$, is less than or equal to $c_2 = 5$, for different $p = 0, 0.01, ...,$

$$\begin{aligned} P_a^{II} &= \{d_1 + d_2 \leq c_2\} \\ &= \sum_{d_1=c_1+1}^{c_2} \left(\frac{n_1!}{d_1!(n_1-d_1)!} p^{d_1}(1-p)^{n_1-d_1} \left(\sum_{d_2=0}^{c_2-d_1} \frac{n_2!}{d_2!(n_2-d_2)!} p^{d_2}(1-p)^{n_2-d_2} \right) \right) \\ &= \sum_{d_1=2}^{6} \left(\frac{50!}{d_1!(150-d_1)!} p^{d_1}(1-p)^{150-d_1} \left(\sum_{d_2=0}^{5-d_1} \frac{100!}{d_2!(100-d_2)!} p^{d_2}(1-p)^{100-d_2} \right) \right) \end{aligned}$$

For example[2], at $p = 0.03$,
$P_a^{II} =$ (choose one) **0.15848 / 0.23011 / 0.28858**

(d) *Probability of acceptance, combined sample*
The probability of acceptance, then, is

$$P_a = P_a^I + P_a^{II}$$

For example, at $p = 0.03$,
$P_a =$ (choose one) **0.15848 / 0.23011 / 0.28858**

[2]The P_a^{II} appears as "ocdiff" on the SAS output.

2. *Sequential sampling: books*
A lot of 3500 books arrives at a bookstore. To check if the entire lot is acceptable, a *sequential sampling plan* is undertaken, where $p_1 = 0.02$, $\alpha = 0.01$, $p_2 = 0.05$ and $\beta = 0.10$.

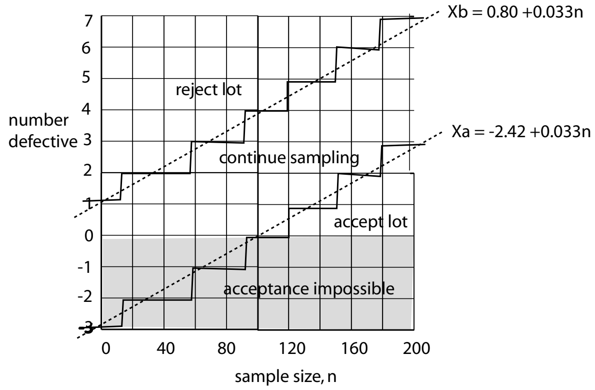

Figure 13.4 (Sequential sampling: books)

(a) *A first look*
From SAS, the limit lines are

$$\begin{aligned} X_A &= -h_1 + sn = -2.41986 + 0.032817n \\ X_B &= h_2 + sn = 0.80406 + 0.032817n \end{aligned}$$

However, instead of X_A, calculate the next *integer* number less than or equal to X_A; and instead of X_B, calculate the next *integer* number greater than or equal to X_B. Then, if $n = 140$,

i. Ac = 2; acceptance number,
accept sample, if number of defectives are less than or equal to 2,
since $X_A = -2.41986 + 0.032817(140) \approx$ (choose one) **0 / 1 / 2**

ii. continue sampling,
if number of defectives are in (2,6],

iii. Re = 6; rejection number
reject sample, if number of defectives are greater than four (6)
since $X_B = 0.80406 + 0.032817(140) \approx$ (choose one) **5 / 6 / 7**

(b) *Impossible acceptance region*
According to this sequential sampling plan it is impossible to accept a lot until the sample size is at least as large at
(choose one) $\boldsymbol{n} = $ **20 / 75 / 130**

(c) *Calculating the limit lines*
True / **False**

$$\begin{aligned}
k &= \log \frac{p_2(1-p_1)}{p_1(1-p_2)} = \log \frac{0.05(1-0.02)}{0.02(1-0.05)} = 0.41144 \\
h_1 &= \left(\log \frac{1-\alpha}{\beta}\right) \div k = \left(\log \frac{1-0.01}{0.10}\right) \div 0.41144 = -2.41986 \\
h_2 &= \left(\log \frac{1-\beta}{\alpha}\right) k = \left(\log \frac{1-0.10}{0.01}\right) (0.41144) = 0.80406 \\
s &= \left(\log \frac{1-p_1}{1-p_2}\right) \div k = \left(\log \frac{1-0.02}{1-0.05}\right) \div 0.41144 = 0.032817
\end{aligned}$$

14.4 Military Standard 105E (ANSI/ASQC Z1.4, ISO 2859)

The standard sampling plan, the MIL STD 105E plan, is discussed in this section.

Exercise 13.3 (Military Standard 105E (ANSI/ASQC Z1.4, ISO 2859))
A lot of $N = 3500$ books arrives at a bookstore. An acceptable quality level (fraction defective) is specified as AQL = 1.5% and the general inspection level is designated as level II (normal).

1. *Sample size code*
   From table 14–4, page 708, the sample size code is
   (choose one) **J** / **L** / **M**

2. *Tightened inspection*
   From table 14–6, page 710, for tightened inspection,
   down one letter to M and so (choose one)

   (a) $n = 315$; Ac = 0, Re = 1

   (b) $n = 315$; Ac = 0, Re = 2

   (c) $n = 125$; Ac = 0, Re = 1

3. *Normal inspection*
   down one letter to M and so (choose one)

   (a) $n = 315$; Ac = 0, Re = 1

   (b) $n = 315$; Ac = 0, Re = 2

   (c) $n = 125$; Ac = 0, Re = 1

4. *Reduced inspection*
   down one letter to M and so (choose one)

   (a) $n = 315$; Ac = 0, Re = 1

   (b) $n = 315$; Ac = 0, Re = 2

   (c) $n = 125$; Ac = 0, Re = 1

5. *MIL STD 105E plan, single and double sampling plan*
   The MIL STD 105E plan in tables 14–5, 14–6 and 14–7, is a *single* sampling plan because all the Ac and Re in these three tables are separated by *one* unit. The lot is either accepted or rejected; it is not possible to continue sampling. For example, if $n = 315$, Ac = 0 and Re = 1, this is equivalent to the single sampling plan (choose one)

   (a) $n = 315$, $c_1 = 0$

   (b) $n = 315$, $c_1 = 1$

   (c) $n = 125$, $c_1 = 2$

   The MIL STD 105E plan can be extended to double and multiple sampling plans.

14.5 The Dodge–Romig Sampling Plans

One standard sampling plan, the Dodge–Romig plan, is discussed in this section.

Exercise 13.4 (The Dodge–Romig Sampling Plans)
A lot of $N = 3500$ books arrives at a bookstore. The *lot tolerance percent defective* (LTPD) is required to be 1.0% and the process average is specified as 0.10%. The single sampling plan required to meet these criteria include,

1. $n = 375$, $c = 1$, AOQL = 0.20

2. $n = 475$, $c = 1$, AOQL = 0.30

3. $n = 575$, $c = 1$, AOQL = 0.20